"tribes living in tropical and hot climates mostly possess the vigesimal system of notation, which is rather infrequent among the Indians of the United States."[1] He finds the explanation in the fact that they live barefoot as contrasted with the moccasined northern Indians. But this "barefoot" explanation, also given by other writers (especially to account for the unusually well developed vigesimal systems of the Mexicans and Aztecs),[2] rather breaks down in the case of the Eskimo and tribes of the north Pacific Coast where the climate is scarcely adapted to the barefoot stage. And yet among the Eskimo the vigesimal system has found its fullest development north of Mexico. The use of fingers and toes for the development of a vigesimal system seems to be independent of climate.

Twenty is the primary basis of the vigesimal system and is usually expressed as "man," "Indian," "man completed"; the multiples of twenty being expressed as "two men," "three Indians," etc. Pawnee carries this to 1,000 which is "50 persons."[3] The vigesimal system usually occurs in connection with some other system. Three types of combination may be noted.

(*a*) *Quinary-vigesimal.* This is most frequent. The Greenland Eskimo says "other hand two" for 7, "first foot two" for 12, "other foot two" for 17 and similar combinations to 20, "man ended." The Unalit is also quinary to twenty which is "man completed." But 40 is "two sets of animals' paws," 60 "three sets of animals' paws" and so on regularly to 400 where there is an interesting change in the formation of this primary base (20×20) from animals back to man, for 400 is "20 sets of *man's* paws."[4]

(*b*) *Decimal-vigesimal.* Systems in which no quinary elements are found are comparatively rare. Wintun is alternately decimal and vigesimal, 20, 40, 60, being "one Indian," "two Indian," "three Indian," while 30, 50 are "3×10," "5×10."[5] Others are similar.

(*c*) *Quinary-decimal-vigesimal.* Several systems show a combination of the three digital bases in their formation. Kopiagmiut is quinary to ten, decimal for the formation of the odd tens and vigesimal for the even ones.[6] Amador is purely decimal to ten, quinary from ten to twenty, and then vigesimal, the odd tens being formed by addition of ten to the preceding even ones, e. g., $50 = 40 + 10$.[7] Haida is quinary-decimal and quinary-vigesimal alternately to a hundred, then pure vigesimal, 400 being $20 \times 20 \times 1$ and 800 being $20 \times 20 \times 2$.[8]

4. **Quaternary Systems.** Fairly well defined quaternary systems reaching

[1] Gatschet, A. S.: "Indian Numeral Adjectives," in *Amer. Antiquarian*, Vol. 2, p. 210.

[2] The best developed vigesimal system on the American continent is probably the Maya of Mexico. This is given and discussed in Conant's *Number Concept* and in much fuller form by Thomas in the 19th Rpt. Bur. of Eth., p. 853.

[3] Dunbar, J. B.: *The Pawnee Language*, 1893. (No place of publication given. Pamphlet, Copy in Newberry Library, Chicago.)

[4] Nelson, E. W.: "Eskimo about Bering Strait," in 18th Ann. Rpt. Bur. of Eth., 1896–7, p. 238.

[5] Dixon & Kroeber: *op. cit.*, p. 675.

[6] Petitot, P.: *Vocabulaire Francaise Esquimau*, Paris, 1876.

[7] Dixon & Kroeber: *op. cit.*, p. 680.

[8] Harrison, C.: "Haida Grammar," in *Royal Soc. of Canada Proc. and Trans.*, Vol. 1, 2d Ser., Sect. 2, p. 123.

to eight may be found among the Montagnais of the far north, the Foxes of Wisconsin, the Iowas and Missouris of the Plains—but they find their best and fullest development into true systems in various California tribes. Usually they are mixed with other systems, but one or two cases are found of practically pure quaternary systems.

It is not easy to account for the origin of such a system. Two possibilities may be suggested. (*a*) *Digital.* Perhaps a few tribes, for reasons best known to themselves, did not use their thumbs in counting. This is possible but there is little if any linguistic or observational evidence to support it. Most frequently these systems show the words "stick," "middle," "body." "Body" might be considered digital. (*b*) *Sacred Number Theory.* Four was the sacred number of many widely separated tribes of Indians. The four cardinal points of the compass and the four seasons were recognized by the Indians. Among several tribes the literal meaning of "four" is "complete," "right," "perfect." However the systems of these tribes are not quaternary ones. A chart for the sacred rites of the Ojibwas shows four degrees of initiation for medicine men. The swastika, with its four arms, originated with the Indians of the Southwest and was much used in basketry. There is a widespread "four worlds" of Indian mythology. In the various languages spoken on Puget Sound the same word for four, with minor variations, is seen most frequently, and it is the only number word common to about a dozen of these languages which have been most carefully studied by a missionary among them. All these data are suggestive of the origin of a quaternary system, although the fact remains that most of the instances given above are from tribes which do not actually possess such systems.

Santa Barbara as far as sixteen is as follows: 1, 2, 3, 4, $X + 1$, $X + 2$, $X + 3$, 8, 9, $X + 2$, 11, 3×4, $X + 1$, $X + 2$, $X + 3$, 16. For 20 and above it is decimal, probably due to contact with civilization. San Luis Obisbo has $X + 2$, $X + 3$, for 6, 7; $8 + 1$ for 9; $12 + 1$, $12 + 2$, $12 + 3$ for 13, 14, 15.[1] Numerous other languages have many quaternary forms.

5. **Ternary System.** A ternary system is much rarer than a quaternary and nowhere occurs in a pure form. Two well-developed ones are given below. It is even more difficult than in the case of the quaternary system to account for its origin. It is possible that to some primitive minds it seemed natural to count one, two, three, and then by groups of threes. In some languages there is a similarity between the words for "this," "that," "that (remote)" or "here," "there," "yonder" and the first three numerals. A number of instances of 6 expressed as 2×3 have already been given[2] and some writers have mentioned them as examples of a ternary system. But for reasons already explained they are better considered as formed by the Duplicative principle, unless occurring in connection with nine or twelve similarly formed. The Cuchan numerals for 3, 6, 9, are ha-mook, hum-hook, hum-ha-mook.[3] In San Antonio 4 is related to 1, 6 is derived from 3, and 12 is directly 4×3, 15 is 5×3, and 13 is $12 + 1$.[4]

[1] Dixon & Kroeber: *op. cit.*, p. 682.
[2] See "Duplicative Principle" already discussed.
[3] Trumbull: *op. cit.*, p. 41.
[4] Dixon & Kroeber: *op. cit.*, pp. 683, 690.

But Coahuiltecan of Texas is the most interesting example found. It has binary, ternary, quaternary, quinary, decimal and vigesimal features,[1] but seems to be prevailingly ternary. Its system as far as 50 is as follows:

1	4	$7 = 4 + 3$	$10 = 5 \times 2$
2	5	$8 = 4 \times 2$	$11 = 10 + 1$
$3 = 2 + 1$	$6 = 3 \times 2$	$9 = 4 + 5$	$12 = 4 \times 3$
$13 = 12 + 1$	$16 = 15 + 1$	$19 = 18 + 1$	$30 = 20 + 10$
$14 = 12 + 2$	$17 = 15 + 2$	20	$40 = 20 \times 2$
$15 = 5 \times 3$	$18 = 6 \times 3$?		$50 = 40 + 10$

6. **Octonary System.** A single system with a base eight is known, the Yuki of California. This interesting system in translation is:[2]

1	9 = beyond-1-hang	17 = 1-middle-project
2	10 = beyond-2-body	18 = 2-middle-project
3	11 = -3-body	19 = 3-middle-project
4	12 = -4-body	20 = 4-middle-project
5	13 = -5-body	
6	14 = -6-body	
7	15 = -7-body	
8	16 = middle-none	

7. **Traces of Other Systems.** Only traces of the use of other bases are found. (*a*) *Binary.* If we include the large number of examples of the duplicative principle already discussed, we have many instances of binary elements. But these are only in the multiplicative principle and refer simply to doubling. The only place where we have found the additive principle used is in the Coahuiltecan (just given) where $3 = 2 + 1$. We may also notice Chutsinni: 2, *stunga;* 4, *stung-sung;* 8, *stun-sunga.* Yokaia: 2, *ko;* 4, *duo-ko;* 8, *ko-ko-dol.* (*b*) *Sexanary.* Aside from the instances of twelve expressed as 2×6 mentioned under the duplicative principle, we have Wimunche Ute: $7 = 6 + 1$, Rumsen: $7 = X + 6$, $8 = 2 + 6$. (*c*) *Base of Nine.* Trinity: $10 = 9 + 1$, $11 = 9 + 2$. (*d*) *Base of Forty.* Chwachamaju: 40, *ku-hai,* "1-stick"; 80, *ko-hai,* "2-stick." (*e*) *Base of Sixty.* Achomawi: $70 = 60 + 10$, $80 = 60 + 20$.

8. **Conclusions.** The most striking feature of the systems which have been studied is their diversity, even in languages of the same family, and much more marked when the country as a whole is considered. For instance in the closely related languages of the Yukian family in California, although the numerals from one to four are quite similar, yet two of the systems are quinary-decimal, a third is quinary-vigesimal, while the fourth is octonary; or in the Pujunan family in which one system is decimal, eight quinary-decimal and two quinary-vigesimal.

How many unique abstract number words are necessary for building up a number system? We recall that in English ten are used and that all up to one hundred are but combinations of these ten. Of course two are sufficient—in a binary system. Many of the Eskimo tribes manage quite well with five. The Luiseno of California has but five abstract number words, but it has higher units

[1] Gallatin, A.: *Trans. Amer. Ethnological Soc.,* Vol. 1, N. Y., 1845, p. 1.

[2] Dixon & Kroeber: *op. cit.,* pp. 677, 685. This system is given by these authors as quaternary, but they have acknowledged that this is an error, and that it should be classed as octonary.

which are chiefly descriptive phrases indicating various combinations of hands and feet. Many languages which have a decimal system get along easily without the full quota of ten as used in English. This is accomplished by the use of such combinations as $8 = 2 \times 4$, $6 = 2 \times 3$, 7, 8, 9 as 3, 2, 1 respectively subtracted from 10, and others which have already been given. The Coahuiltecan (given under the ternary system) forms all the numbers up to twenty with only four unique words, those for 1, 2, 4, 5—a very remarkable instance.

III. Miscellaneous Points.

1. **Limits in Use.** The numerals in some languages are given as high as a million and in many others to a thousand or more. We are naturally led to inquire whether such high numbers were actually in use by primitive people. The evidence found on this point cannot be here given in detail. Only a few conclusions may be stated. The Eskimo seem to be poorest in ability to count, being low in comparison with the tribes of the United States. Most of them in ordinary conversation do not use above five or six, referring to higher numbers as "many," but the more intelligent of them can count to 400. The Indians of the eastern United States who stand comparatively high intellectually, could use their numerals to 10,000 and their systems were such as to admit of indefinite expansion, one billion for instance being expressed as $1{,}000 \times 1{,}000 \times 1{,}000$. The Crees could count correctly as far as 1,000. The Winnebagoes are said to use their numerals as high as one million Indefinite and countless numbers they represent by the terms "leaves on the trees," "stars of the heavens," "blades of grass on the prairie," "sand on the lake shore." The Crows do not count above a thousand, as they say honest people have no use for higher numerals! The Apaches cannot use numbers beyond 100,000. Any of the California tribes of which positive statements can be made can count into the hundreds. As a definite upper limit to counting ability we find that the Tuolomne are credited with the ability "to count with great rapidity almost to infinity"![1]

Speaking in general terms, we may say that the rather highly developed Siouan tribes of the Plains, the Iroquoian and Algonquian families of the east, and the Muskogean tribes of the South—all rather high in the scale of civilization—could count intelligently at least into hundreds of thousands and had words for even higher numbers; that most of the other Indians of the country had little actual knowledge of forms for numbers higher than thousands; while the Eskimo of the far north were limited to hundreds and in many cases to twenty or even ten. It is probable that before coming in contact with European civilization the Indians had little occasion to use numbers beyond a thousand. But the systems of many of them were such as to admit of indefinite and easy extension when needed.

2. **Numeral Classifiers.** Some tribes use different sets of numerals for counting different classes of objects. The Tsimsian language has quite distinct forms for counting *men* and for counting *things*. More frequently the number stem is

[1] Johnson, A.: In Schoolcraft's *Indian Tribes* (*op. cit.*), Vol. IV, p. 406.

modified by a prefix or suffix which is in the nature of a classifier to denote the class of objects counted. The Haida has no less than 15 of these classifiers. Others seem to have an even larger number.[1] This usage is found very generally among the languages of the north Pacific coast and but rarely in other parts of the country. The Tsimsian mentioned above has different forms for abstract counting, for counting flat objects or animals, for round objects or time, for men, for long objects, for canoes, for measures.

3. **Verbal Nature.** In a few languages the numerals are true verbs instead of adjectives, and as such are conjugated through all the variations of mood, tense, person and number. As far as found this peculiarity is limited to three languages, Cree, Crow, Micmac.

4. **Derivative Numerals.** The formation of ordinals, adverbials and distributives from the cardinal numerals is more properly a grammatical than a numerical process and as such need receive only slight notice here. (*a*) *Ordinals* are found most frequently, usually being formed from the cardinals by a suffix or other terminal modification, occasionally by a prefix. In the Creek an unusual method is found, the ordinal being formed from the cardinal in the same way that the superlative of the adjective is formed from the comparative. (*b*) *Distributives* are also frequently found. They are formed from the cardinals by prefixes, suffixes, or reduplication. (*c*) *Adverbials*, as far as noted, are formed by suffixes.

5. **Arithmetical Operations.** We shall close our discussion of the varied, interesting, and intricate number systems of the North American Indians with a reference to their ability to perform arithmetical operations. In our study of the principles of formation of number words we found an extensive use of addition and multiplication, a less use of subtraction, and very slight use of division in the formation of number systems. But aside from these instances (all operations on only the *bases* of the systems) the calculative ability of the American Indians was very slight and of the most elementary sort. Addition, subtraction or multiplication was accomplished only with the aid of the fingers, sticks, pebbles or other convenient counters. It is probable that the native Indian mind had practically no idea of mental arithmetic, being unable to multiply or divide numbers mentally, or even to add or subtract any except the smallest. His need for such operations was probably as slight as his knowledge.

THE CURVE OF LIGHT ON A CORRUGATED DOME.

By WM. H. ROEVER, Washington University.

The Phenomenon. On a bright day a person situated at a high point, within a distance of three or four miles of the new Roman Catholic Cathedral of St. Louis, will observe a well-defined curve of light on the dome of the cathedral. This curve, which is closed, passes through the highest point of the dome and

[1] The Maya of Mexico has at least 75 such classifiers. See article by Mrs. Z. Nuttall in *Amer. Anthropologist*, N.S., Vol. 5, p. 667.

changes its size and position with the changing position of the sun. The accompanying photograph (Fig. 1), taken from a high building about a mile from the Cathedral, shows this closed curve which resembles a halo. The same sort of phenomenon may be observed on the stem of a watch when held in the light.

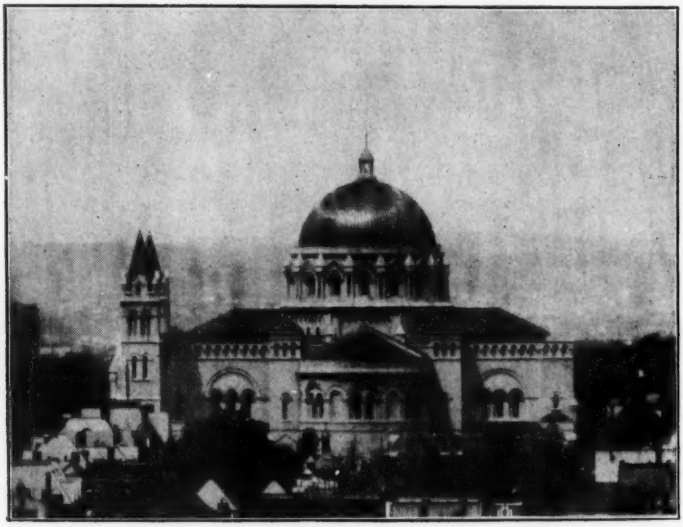


FIG. 1.

The Cause. The dome of the cathedral is covered with glazed tiles laid along those great circles of the spherical dome which pass through the highest point, thus forming a sort of corrugated surface, of which the ridges lie

FIG. 2.

along the meridians of the sphere (Fig. 2). The sun's light is reflected from this glazed, corrugated surface, and an observer, properly situated, sees in each corrugation a reflected image of the sun or an actual brilliant point, as this image is sometimes called. The assemblage of these images or brilliant points, any two consecutive ones of which are too near each other to appear separated to the eye of the distant observer, forms the curve or halo described above.

The Mathematical Problem Involved. In order to determine the mathematical nature of this curve of light we must make some assumptions, as we do in the mathematical formulation of every physical problem. It is assumed that the dome is a portion of a spherical surface, in fact, a hemisphere, and that the meridional corrugations are so small and numerous that we may regard the spherical dome as covered with a family of polished wires which are laid along the great circles through the highest point. It is further assumed that these wires are so fine that they may be regarded as mathematical curves. Furthermore, the sun's rays are regarded as a system of parallel rays, and the observer is supposed to be far enough away from the Cathedral so that the reflected rays which reach his eye may also be regarded as belonging to a system of parallel rays. Under these assumptions the mathematical problem involved may be stated as follows:

Determine the locus of the actual brilliant points of the meridians of a sphere with respect to a source of light which is infinitely distant and an observer who is (practically) infinitely distant.

Solution of the Mathematical Problem.

Definition. A point P is said to be an *actual brilliant point* of the curve C (plane or twisted) with respect to the source of light P_1 and an observer P_2 when

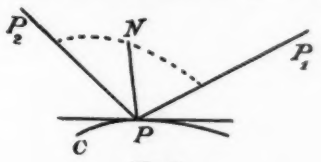


Fig. 3.

the internal bisector of the plane angle P_1PP_2 is a normal to the curve C at the point P. In particular P_1 and P_2 may be infinitely distant.

Accordingly, if we denote by l_1, m_1, n_1 the direction cosines of the directed line PP_1, by l_2, m_2, n_2 the direction cosines of the directed line PP_2, and by l, m, n numbers proportional to the direction cosines of the tangent line to the curve C at the point P, then the condition that the point P should be an actual brilliant point of the curve C, with respect to the source P_1, and the observer P_2, is*

$$(1) \qquad (l_1 + l_2)l + (m_1 + m_2)m + (n_1 + n_2)n = 0.$$

* For more details see the author's paper "Bril ant Points of Curves and Surfaces," *Transactions of the American Mathematical Society*, Vol. 9, No. 2, pp. 245–279.

In our particular problem the source P_1 is the infinitely distant sun, P_2 is the eye of the (practically) infinitely distant observer and the curve C is a vertical great circle, that is a meridian, of the spherical dome. In order to get the form of condition (1) for our problem, let us assume a set of rectangular axes of which the origin lies at the center of the spherical dome and of which the axis of z is vertical and positive in the direction of the zenith. The equations of a meridian then are (Fig. 4)

$$F = 0 \quad \text{and} \quad \Phi = 0, \tag{2}$$

where

$$F = x^2 + y^2 + z^2 - R^2 \quad \text{and} \quad \Phi = \frac{y}{x} - k,$$

in which R denotes the radius of the spherical dome and k is a parameter whose value depends on the particular meridian under consideration.

The direction cosines of a tangent to the curve (2) at a point (x, y, z) of this curve are proportional to the three determinants of the matrix

$$\left\| \begin{matrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial \Phi}{\partial x} & \frac{\partial \Phi}{\partial y} & \frac{\partial \Phi}{\partial z} \end{matrix} \right\|.$$

Hence if, as above, we denote by l, m, n numbers proportional to these direction cosines, we have

$$l : m : n = -xz : -yz : x^2 + y^2. \tag{3}$$

Since, for our problem, l_1, l_2, m_1, m_2, n_1, n_2 are constants at any particular instant of time, let us introduce new constants by the relations:[1]

$$\alpha = l_1 + l_2, \quad \beta = m_1 + m_2, \quad \gamma = n_1 + n_2.$$

Hence condition (1) for our problem takes the form:

$$-\alpha xz - \beta yz + \gamma(x^2 + y^2) = 0. \tag{4}$$

This equation represents a quadric cone whose vertex is at the origin of coördinates, and whose horizontal sections are circles. This cone contains as two of its elements the axis of z and the internal bisector of the angle formed at the origin by the rays which pass to the sun and to the observer, and the plane of these two elements is a plane of symmetry. It is the locus of all of the actual brilliant points of the two parameter family of curves obtained by allowing both R and k in equations (2) to be variable parameters. The locus which we are seeking is the intersection of this locus with the sphere:

$$x^2 + y^2 + z^2 - R^2 = 0. \tag{5}$$

Since the dome is the upper hemisphere of the sphere (5), we are interested only

[1] These constants are proportional to the direction cosines of the internal bisector PN of the angle P_1PP_2.

in the upper branch of the curve made by the intersection of (4) and (5). This is the only portion of the locus which we observe. Figure 4 is an orthographic projection in which the sun is represented as being in the xz plane at an altitude of 60° and the observer at an infinite distance in a direction perpendicular to the plane of the drawing.

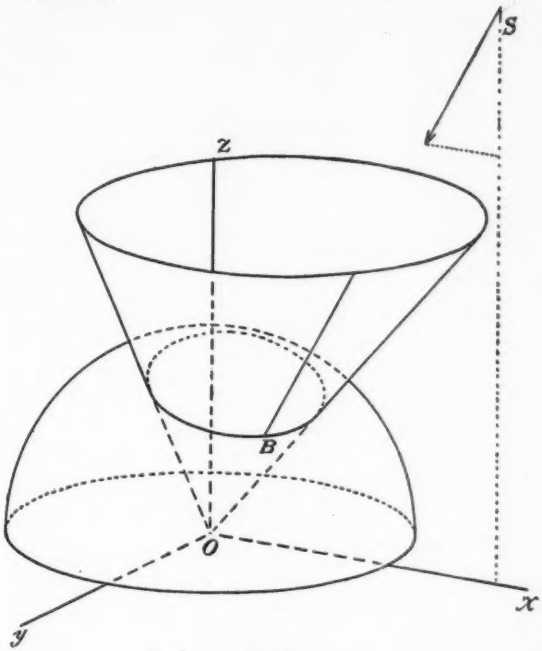


Fig. 4.

The observer is practically in the horizontal plane which passes through the center of the dome. Hence, practically, $n_2 = 0$. When the sun is in the horizon $n_1 = 0$ also, and therefore $\gamma = 0$. Then equation (4) becomes

$$z(\alpha x + \beta y) = 0,$$

and the curve of light (4)(5) degenerates into two great circles of the sphere, of which one is horizontal and the other vertical.

Finally, it is interesting to note that if the corrugations had been parallels instead of meridians, the curve of light would have been a meridian.[1]

Figure 1 shows, in addition to the curve of light on the dome, a line of light on the conical roof of the eastern apse of the Cathedral. This is the element along which a plane perpendicular to the internal bisector of the angle P_1PP_2 (Fig. 3) is tangent to the conical roof.

[1] This will be further elucidated in paper on "Optical Interpretations in Higher Geodesy" soon to be published in the MONTHLY.

ON CERTAIN DIOPHANTINE EQUATIONS HAVING MULTIPLE PARAMETER SOLUTIONS.[1]

By R. D. CARMICHAEL, Indiana University.

1. The object of this note is to obtain a multiple parameter solution of each of several Diophantine equations, as follows:

$$\begin{aligned} x^3+y^3+z^3&=2t^3, & x^3+y^3+z^3&=2t^{9k},\\ x^3+y^3+z^3+u^3&=3t^3, & x^3+y^3+z^3+u^3&=kt^a,\\ x^3+2y^3+3z^3&=t^3, & x^3+2y^{3\alpha}+3z^{3\beta}&=t^3,\\ x^4+y^4+4z^4&=t^4, & x^4+2y^4+2z^4&=t^4,\\ x^4+8y^4+8z^4&=t^4, & x^4+y^4+z^4&=2t^4, \end{aligned}$$

where k is a positive integer and α and β are odd positive integers. The methods employed are altogether elementary in their character.

2. If in the equation[2]

$$x^3+y^3+z^3=2t^3, \tag{1}$$

the numbers x, y, z, t have the common divisor d, then the equation may be divided through by d^3 and the resulting equation will be of the same form as (1). Hence it is sufficient to seek only those solutions of (1) in which x, y, z, t have the greatest common divisor unity; and consequently we confine attention to this case. It is obvious, then, that two of the numbers x, y, z, are odd while the third one is even. If we suppose x and y to be odd, we may write

$$x=r+s, \quad y=r-s,$$

where r and s are integers.

Substituting these values of x and y in (1) we have

$$2r^3+6rs^2=2t^3-z^3.$$

This equation will be satisfied if we have simultaneously $r=t$, $z=-6rs^2$. If we write r in the form $r=k\rho^3$ where k has no cube factor except unity, then we must have $s=\pm k\sigma^3$. It is obvious that in the resulting solution the numbers x, y, z, t have the common factor k; and hence, for our present case, we have $k=1$. Our desired solution of (1) then is

$$x=\rho^3\pm 6\sigma^3,\ y=\rho^3\mp 6\sigma^3,\ z=-6\rho\sigma^2,\ t=\rho^3. \tag{2}$$

It should be noticed that there is nothing in the argument to indicate whether (2) affords the general solution of (1). A similar remark applies to the other problems treated in this note.

It may be observed in passing that we have incidentally obtained a two-

[1] Presented to the American Mathematical Society, March 21, 1913.

[2] A single solution of this equation is given in the French Encyclopédie, tome I, vol. 3, p. 210; namely, $x=7$, $y=11$, $z=102$, $t=81$. This is not a special case of the solution obtained in the text.

parameter solution of the equation

$$x^3 + y^3 + z^3 = 2t^9,$$

x, y, z having the same values as before and t being equal to ρ. By taking ρ equal to the kth power of an integer we have further a two-parameter solution of the equation

$$x^3 + y^3 + z^3 = 2t^{9k}.$$

3. We shall now obtain solutions of the equation

$$x^3 + y^3 + z^3 + u^3 = 3t^3. \tag{3}$$

If we write

$$x = a + b - c, \quad y = a - b + c, \quad z = -a + b + c,$$

we have

$$a = \tfrac{1}{2}(x + y), \quad b = \tfrac{1}{2}(x + z), \quad c = \tfrac{1}{2}(y + z),$$

so that a number of the set a, b, c either is an integer or is the half of an odd integer. Substituting these values of x, y, z in (3) we obtain

$$(a + b + c)^3 - 24\,abc + u^3 = 3t^3.$$

This equation will be satisfied if we have simultaneously $u = -(a + b + c)$, $t^3 = -8abc$. If we write $a = m\alpha^3$, $b = n\beta^3$, $mn = ks^3$, where neither m nor n nor k has a cubic factor greater than 1, we have $c = \pm k^2\gamma^3$. Therefore we have the following five-parameter solution of (3)

$$x = m\alpha^3 + n\beta^3 \pm k^2\gamma^3, \quad y = m\alpha^3 - n\beta^3 \pm k^2\gamma^3, \quad z = -m\alpha^3 + n\beta^3 \pm k^2\gamma^3, \tag{4}$$

$$u = -(m\alpha^3 + n\beta^3 \pm k^2\gamma^3), \quad t = \pm 2ks\alpha\beta\gamma,$$

where $mn = ks^3$. The parameters entering into the solution represent either integers or the halves of odd integers, the latter being so related that the values of x, y, z, u, t are all integers.

It is clear that we may also readily obtain a five-parameter solution of the equation

$$x^3 + y^3 + z^3 + u^3 = kt^n,$$

where α is any positive odd integer and k is any integer.

4. In the equation

$$x^3 + 2y^3 + 3z^3 = t^3, \tag{5}$$

let us make the substitution

$$x = r - s, \quad t = r + s;$$

then we have $2y^3 + 3z^3 = 2s^3 + 6r^2s$. This will be satisfied if we have simultaneously $y = s$, $z^3 = 2r^2s$. We exclude the case when r and s have a common factor greater than unity, since this would lead to a solution of x, y, z, t with such a common factor. Then it is clear that we have $s = m^3$, $r = 2n^3$. Hence we obtain the two-parameter solution

$$x = 2n^3 - m^3, \quad y = m^3, \quad z = 2mn^2, \quad t = 2n^3 + m^3. \tag{6}$$

It is easy to see that we have obtained incidentally a two-parameter solution of the equation

$$x^3 + 2y^{3\alpha} + 3z^{3\beta} = t^3, \tag{7}$$

where α and β are any odd positive integers. For, let μ be the least common multiple of α and β and write

$$m = r^\mu, \quad 2n^2 = s^\mu = (2\cdot\sigma^2)^\mu,$$

whence $n = 2^{\frac{\mu-1}{2}}\sigma^\mu$. These values of m and n, taken in connection with (6), lead to the following solution of (7):

$$x = 2^{\frac{3\mu-1}{2}}\sigma^{3\mu} - r^{3\mu}, \quad y = r^{\frac{3\mu}{\alpha}}, \quad z = 2^{\frac{\mu}{\beta}}\sigma^{\frac{2\mu}{\beta}}r^{\frac{\mu}{\beta}}, \quad t = 2^{\frac{3\mu-1}{2}}\sigma^{3\mu} + r^{3\mu}. \tag{8}$$

As an analogue of equation (5) we have the equation

$$x^2 + 2y^2 = z^2$$

with the solution

$$x = 2t^2 - s^2, \quad y = 2st, \quad z = 2t^2 + s^2.$$

5. If for the equation

$$x^4 + y^4 + 4z^4 = t^4 \tag{9}$$

we confine attention to the case in which x, y, z, t have the greatest common divisor 1, it is easy to see that t is odd and either x or y is odd. We shall suppose that x is odd. Then if we write

$$x = r - s, \quad t = r + s,$$

it follows that r and s are integers. Substituting these values in (9) we have

$$y^4 + 4z^4 = 8r^3s + 8rs^3.$$

This equation will be satisfied if we have simultaneously $y^4 = 8r^3s$, $4z^4 = 8rs^3$ or $y^4 = 8rs^3$, $4z^4 = 8r^3s$. It is clear that these two cases will not lead to different solutions of (9).

Let us consider the first case: $y^4 = 8r^3s$, $4z^4 = 8rs^3$. If $r = k\rho^4$, where k has no fourth power factor except unity, then $s = 2k\sigma^4$, as we see from the first equation. Hence $4z^4 = 64k^4\rho^4\sigma^{12}$. If x, y, z, t are to have no common factor (except unity) it is clear that $k = 1$. Hence we are led to the following solution of (9):

$$x = \rho^4 - 2\sigma^4, \quad y = 2\rho^3\sigma, \quad z = 2\rho\sigma^3, \quad t = \rho^4 + 2\sigma^4. \tag{10}$$

6. Proceeding in a similar way, we obtain for the equations

$$x^4 + 2y^4 + 2z^4 = t^4,$$
$$x^4 + 8y^4 + 8z^4 = t^4,$$
$$x^4 + y^4 + 2z^4 = 2t^4,$$
$$x^4 + y^4 + 8z^4 = 8t^4,$$

the solutions

$$\begin{array}{llll} x = 4\rho^4 - \sigma^4, & y = 4\rho^3\sigma, & z = 2\rho\sigma^3, & t = 4\rho^4 + \sigma^4, \\ x = \rho^4 - \sigma^4, & y = \rho^3\sigma, & z = \rho\sigma^3, & t = \rho^4 + \sigma^4, \\ x = 2\rho^3\sigma, & y = 2\rho\sigma^3, & z = \rho^4 - \sigma^4, & t = \rho^4 + \sigma^4, \\ x = 8\rho^3\sigma, & y = 4\rho\sigma^3, & z = 4\rho^4 - \sigma^4, & t = 4\rho^4 + \sigma^4, \end{array}$$

respectively.

7. Let us consider next the equation

$$x^4 + y^4 + z^4 = 2t^4. \tag{11}$$

We shall seek solutions of this equation in which x, y, z, t are polynomials in an integral parameter k, the degree being not greater than 2. It is easy to see that the coefficients of k^2 in these polynomials must satisfy equation (11). The corresponding statement is true also of the independent terms. Now (11) has the obvious solutions obtained by putting one of the variables in the first member equal to zero and giving to the other variables in any way the values $+1$ or -1. This gives us a certain number of cases to examine in order to see whether the coefficients of powers of k in the several polynomials can be so determined that (11) becomes an identity on substituting for the variables the polynomials in k.

Thus, as one case, we have to examine whether the ambiguous signs can be suitably chosen and the coefficients a, b, c, d so determined that

$$(k^2 + ak)^4 + (bk \pm 1)^4 + (k^2 + ck \pm 1)^4 = 2(k^2 + dk \pm 1)^4$$

is an algebraic identity in k. Carrying out the necessary reckoning in each of the possible cases which may arise we have the following solution of (11):

$$x = k^2 - 2k, \quad y = 2k - 1, \quad z = k^2 - 1, \quad t = k^2 - k + 1. \tag{12}$$

The reckoning is facilitated by observing that when k is replaced by $k + 1$ the resulting independent terms must still form a solution of (11).

THE CUBE ROOT OF A BINOMIAL SURD.

By ARTHUR C. JOHNSON, Hopedale, Mass.

In Wells's *Text Book in Algebra*, page 548, the equation $x^3 + x - 2 = 0$ leads by Cardan's method to the solution $\frac{1}{3}(\sqrt[3]{27 + 6\sqrt{21}} + \sqrt[3]{27 - 6\sqrt{21}})$, and the author implies that this cannot be simplified, for he says "there is no method in algebra for finding the cube root of a binomial surd."

The challenge suggested by this problem led to the following discussion, which, so far as the writer can learn, has not been published heretofore. Namely, to determine whether a quadratic binomial surd has a cube root which can be obtained by algebraic methods; and, when possible, to obtain this root.

Given the quadratic binomial surd $a + b\sqrt{c}$ in which a, b and c are positive

and rational, and $\sqrt{c}$ is a positive quadratic surd. We assume that the cube root of this expression, if obtainable, is a binomial of which one term x is positive and rational; the other term y is a positive quadratic surd. Thus

$$x + y = \sqrt[3]{a + b\sqrt{c}} \tag{1}$$

Then

$$x - y = \sqrt[3]{a - b\sqrt{c}}, \tag{2}$$

by the method shown in Section 269 of Wells's text. Hence

$$x^2 - y^2 = \sqrt[3]{a^2 - b^2c}. \tag{3}$$

Now since $x^2 - y^2$ is rational, the expression $\sqrt[3]{a^2 - b^2c}$ must also be rational. Consequently when the expression $a^2 - b^2c$ is a perfect cube, it will be possible to use this method.

Cubing (1),

$$x^3 + 3x^2y + 3xy^2 + y^3 = a + b\sqrt{c}. \tag{4}$$

Now the second and fourth terms of (4), in which odd powers of y occur, must be irrational, and the first and third terms must be rational.

Hence

$$3x^2y + y^3 = b\sqrt{c} \tag{5}$$

and

$$x^3 + 3xy^2 = a. \tag{6}$$

Combining (3) and (6), we have

$$4x^3 - 3x\sqrt[3]{a^2 - b^2c} - a = 0. \tag{7}$$

When $\sqrt[3]{a^2 - b^2c}$ is rational, (7) can be solved by the ordinary methods for cubic equations and the corresponding value of y can be found.

Whenever (7) has a commensurable root it is therefore possible to find the cube root of the given binomial surd by algebraic methods in the form of a quadratic binomial surd.

In the example quoted above we have to find the cube root of $27 + 6\sqrt{21}$. In this case, $a = 27$, $b = 6$, $c = 21$.

$$\sqrt[3]{a^2 - b^2c} = \sqrt[3]{729 - 756} = \sqrt[3]{-27} = -3.$$

Substituting this value in (7), the equation becomes

$$4x^3 + 9x - 27 = 0,$$

and

$$x = \frac{3}{2}, \quad y = \frac{\sqrt{21}}{2}.$$

Hence the cube root of $27 + 6\sqrt{21}$ is $\frac{3}{2} + \frac{\sqrt{21}}{2}$, and in like manner the cube root of $27 - 6\sqrt{21}$ is $\frac{3}{2} - \frac{\sqrt{21}}{2}$.

Consequently

$$\frac{\sqrt[3]{27+6\sqrt{21}}+\sqrt[3]{27-6\sqrt{21}}}{3}=1,$$

the result expected from the equation $x^3+x-2=0$.

The method may be extended, and holds when a, b and c are negative as well as positive. Likewise the proof holds when the required root can be found in the form $a\sqrt{b}+c\sqrt{d}$.

The latter expression gives rise to the cubic equation

$$4bx^3-3x\sqrt[3]{a^2b-c^2d}-a=0. \tag{8}$$

Example 1. To find the cube root of $2{,}261\sqrt{2}-1{,}845\sqrt{3}$. Here $a=2{,}261$, $b=2$, $c=-1{,}845$, $d=3$.

$$\sqrt[3]{a^2b-c^2d}=\sqrt[3]{10{,}224{,}242-10{,}212{,}075}=\sqrt[3]{12{,}167}=23.$$

Substituting this value in (8), the equation becomes

$$8x^3-69x-2{,}261=0,$$

and

$$x=7,\quad y=5.$$

Therefore the cube root of $2{,}261\sqrt{2}-1{,}845\sqrt{3}$ is $7\sqrt{2}-5\sqrt{3}$.

Example 2. To find the cube root of $276\sqrt{3}-70\sqrt{-2}$. Here $a=276$, $b=3$, $c=-70$, $d=-2$.

$$\sqrt[3]{a^2b-c^2d}=\sqrt[3]{228{,}528+9{,}800}=\sqrt[3]{238{,}328}=62.$$

Substituting this value in (8), the equation becomes

$$12x^3-186x-276=0,$$

whence $x=-2$, and $y=5$. Consequently the cube root of $276\sqrt{3}-70\sqrt{-2}$ is $-2\sqrt{3}+5\sqrt{-2}$.

BOOK REVIEWS.

W. H. Bussey, Chairman of the Committee.

Trigonometry for Schools and Colleges. By Frederick Anderegg and Edward Drake Roe. Revised edition by Frederick Anderegg. Ginn and Company, Boston, 1913. ix + 108 pages. 75 cents.

The characteristics which insured for this little book a favorable reception in its first edition, namely clearness and conciseness, have been retained in this new edition.

The striking alterations effected in the revision of the plane trigonometry touch the solution of triangles. The right triangle is solved in the third chapter instead of in the sixth and model solutions of both right and oblique triangles

are added. The discussion of the various cases has been somewhat elaborated and the number of numerical exercises increased.

In the more theoretical part of the text there are many minor changes in the direction of brevity, which completely compensate for the additions and leave the number of pages unaltered.

There are no changes in that part of the book which deals with spherical trigonometry.

J. W. BRADSHAW.

An Introduction to the Mathematical Theory of Heat Conduction. By L. R. INGERSOLL and O. J. ZOBEL. Ginn and Co., Boston, 1913. 171 pages.

This is a handy little volume on the Fourier theory, intended partly as an introduction to the general methods of mathematical physics, or rather of those parts which are concerned with boundary-value problems in the integration of partial differential equations, partly to give an account of a number of special problems selected from the wide range of applications in physics, engineering, and geology, as the sub-title suggests. In view of this two-fold aim it is convenient to consider separately the treatment of the general theory and of the special problems.

As one would expect, the general mathematical material introduced coincides in the main with that contained in the more elementary portions of Fourier's own work, with more modern notation and some simplification of arrangement. It covers the more important of those cases of steady and variable distribution of temperature which are solvable by simple formulæ in elementary functions, by ordinary trigonometric series, or by Fourier's integral, either directly or by the device of extension of the realm of definition of a function such as to make certain boundary properties automatic. The problem of the sphere is also included, as solved for the case of spherical isothermals by generalized Fourier series.

In the more purely mathematical portions of the book there are a number of things which seem to call for criticism, a few of which may be named here. Certain infinite series used in leading up to Fourier's integral are not convergent, and the integral itself is written in some places so that the inner integral diverges. In the explanation of the formula of conduction the terms "plane" and "lamina" seem to be used as synonymous. In one place it is stated that the assumption of expansibility of a function into a sine-series is to be considered justified if values can be found for the coefficients. The term "general solution," from which special solutions are to be obtained by "substituting," is used, without any real need, in connection with partial differential equations, where the notion is practically useless and usually disagreeably vague unless taken in the unenlightening sense of the class of all solutions; then the comforting assurance is added that many cases can be solved by other methods, even when it would be "almost impossible" to obtain the general solution of the differential equation. These few instances illustrate an occasional looseness of expression which one could

fairly be expected to avoid without sacrificing the desired condensation of statement to which it seems to be due.

The physical basis of Fourier's fundamental equations is also treated concisely, but space is found for a sufficient discussion of numerical computations and of the various units of measure used. The dimensions assigned to quantity of heat are questionable. In one place it is suggested that conduction in the gross is really an aggregate effect of transfers of heat between molecules, which may occur with the velocity of sound; one wonders what basis there is for this. As in the mathematical portions, such purely incidental and probably misleading remarks, too brief to be both accurate and intelligible, might better have been omitted. Also it is stated that conduction depends only on differences of temperature, the actual temperature being immaterial, then, further on, data are given showing the variation of conductivity with temperature, and in another place it is stated that the use of the mean conductivity for the range of temperature in question gives an approximate solution which may be practically close enough. Here a little more coherence in the arrangement might have made this fundamental matter clearer, and also have served to give needed emphasis to the limitations of Fourier's assumptions.

The chief merit of the book will probably be found in the special problems which are presented as illustrations of the practical bearing and value of the analytic formulas. It is apparent that the main labor of the authors has been devoted to collecting, sifting, and by some novelties, adding to this remarkable list of special applications, whose titles fill a column in the index. They are all of some intrinsic interest, and are worked out briefly, but still far enough to secure some numerically definite conclusions. The extent and variety of this material make the book unique among the works on this subject known to the reviewer. Mention should also be made of the many splendid diagrams, illustrating the convergence of Fourier series, or giving various temperature-curves.

A. C. LUNN.

PROBLEMS AND SOLUTIONS.

B. F. FINKEL, CHAIRMAN OF THE COMMITTEE.

PROBLEMS FOR SOLUTION.

Special Notice.—Please re-read the requests as to form of solutions on pp. 258–259 of the October issue. Unless these directions are observed by contributors, solutions must either be entirely rewritten by the committee or else rejected.

MANAGING EDITOR.

ALGEBRA.

397. Proposed by W. H. BUSSEY, University of Minnesota.

12 oxen are turned into a pasture of $3\frac{1}{3}$ acres and eat all the grass in 4 weeks so that the pasture is bare. 21 oxen are turned into a pasture of 10 acres and eat all the grass in 9 weeks. How many oxen will eat all the grass in a pasture of 24 acres in just exactly 18 weeks, it being assumed that the grass in all the pastures is at the same height when the oxen are turned in, and that the grass grows at a uniform rate?

GEOMETRY.

426. Proposed by R. D. CARMICHAEL, Indiana University.

On a given chord of a circle as a base construct an isosceles triangle, with vertex outside of the circle, such that its sides shall be divided in a given ratio by their points of intersection with the circle.

CALCULUS.

347. Proposed by R. D. CARMICHAEL, Indiana University.

Show that the differential equation

$$9\left(\frac{d^2y}{dx^2}\right)^2\frac{d^5y}{dx^5} - 45\frac{d^2y}{dx^2}\frac{d^3y}{dx^3}\frac{d^4y}{dx^4} + 40\left(\frac{d^3y}{dx^3}\right)^3 = 0$$

remains unchanged when the variables x and y undergo any projective transformation (Goursat-Hedrick, *Mathematical Analysis*, p. 86, ex. 18).

348. Proposed by E. L. DODD, University of Texas.

Let $(x_1, x_2, \cdots x_n)$ be a point in n dimensions lying in the "sphere" S defined by

$$x_1^2 + x_2^2 + \cdots + x_n^2 \leqq 1.$$

Let T be that part of S defined by a set of n linear homogeneous inequalities with non-vanishing determinant, thus:

$$a_ix_1 + b_ix_2 + \cdots + k_ix_n \geqq 0, \quad i = 1, 2, \cdots n.$$

Find the value of

$$\frac{\int_T \cdots \int dx_1 \cdots dx_n}{\int_S \cdots \int dx_1 \cdots dx_n};$$

in other words, find the magnitude of a "solid angle" in n dimensions, with the "sphere" as unit solid angle.

Note.—This problem was discussed and left unsolved by Schläfli in the Quarterly Journal of Mathematics for 1858, 1860, 1867. EDITOR.

349. Proposed by C. N. SCHMALL, New York City.

If $y = a\cos(\log x) + b\sin(\log x)$, eliminate the constants a and b and obtain the equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

MECHANICS.

282. Proposed by R. P. LOCHNER, Philadelphia, Pa.

A car weighing 10 tons (2,240 lb. each) attains a speed of 15 miles per hour from rest in 24 seconds, during which it covers 100 yards. If the space-average of the resistances is 30 lb. per ton, find the average horse-power used to drive the car. (Morley's "*Mechanics for Engineers*," p. 66).

283. Proposed by C. N. SCHMALL, New York, N. Y.

The maximum length of a certain chain which can be suspended from one end without breaking is l. It is desired to form a catenary with a length $2l/k$ of the chain, the points of support being a distance d apart, in the same horizontal line.

Show that the maximum value of d is $\frac{2l}{k}(k^2-1)^{\frac{1}{2}}\log\left(\frac{k+1}{k-1}\right)^{\frac{1}{2}}$.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

199. Proposed by R. P. LOCHNER, Philadelphia, Pa.

Find three integral squares such that the sum of every two of them shall be a square.—Alsop's "Algebra."

200. Proposed by R. D. CARMICHAEL, Indiana University.

Find the general solution, in relatively prime integers, of the equation $x^3 + y^3 = z^4$.

201. Proposed by E. T. BELL, Seattle, Washington.

Eisenstein (*Crelle*, t. 27, p. 282) proposed, as the simplest of several problems: "In the expansion of

$$\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1 - z)^{p-1}} - 1,$$

where p is prime, to show that the coefficients of the various powers of z are all divisible by p."

202. Proposed by A. R. SCHWEITZER, Chicago, Ill.

There exists an infinitude of systems of dyads $\{\alpha\beta\}$ in 7, 9, 11, etc., elements such that each system has the following properties: (1) if $\alpha\beta$ is in the set, then $\beta\alpha$ is not in the set; (2) for each dyad $\alpha\beta$ in the set there exists an element ξ such that $\xi\beta$ and $\alpha\xi$ are also in the set. For example, such a system is,

12,	23,	34,	45,	56,	67,	78,	89,	91
13,	24,	35,	46,	57,	68,	79,	81,	92
14,	25,	36,	47,	58,	69,	71,	82,	93
51,	62,	73,	84,	95,	16,	27,	38,	49

Investigate the existence of

I. A finite set of triads $\{\alpha\beta\gamma\}$ such that (1) if $\alpha\beta\gamma$ is in the set, then $\beta\gamma\alpha$, $\gamma\alpha\beta$ are also in the set but $\beta\alpha\gamma$ is not in the set, (2) for each triad $\alpha\beta\gamma$ in the set there exists an element ξ such that $\xi\beta\gamma$, $\alpha\xi\gamma$, $\alpha\beta\xi$ are also in the set.

II. A finite set of tetrads $\{\alpha\beta\gamma\delta\}$ such that (1) if $\alpha\beta\gamma\delta$ is in the set, then $\beta\gamma\alpha\delta$, $\gamma\alpha\beta\delta$, $\gamma\delta\alpha\beta$ are also in the set but $\beta\alpha\gamma\delta$ is not in the set, (2) for each tetrad $\alpha\beta\gamma\delta$ in the set there exists an element ξ such that $\xi\beta\gamma\delta$, $\alpha\xi\gamma\delta$, $\alpha\beta\xi\delta$, $\alpha\beta\gamma\xi$ are also in the set.

The problem for alternating *n-ads* for $n > 4$ is obvious.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

382. Proposed by C. E. FLANAGAN, Wheeling, W. Va.

A few days ago, I deduced the following formula for finding the value of the unknown quantity in a cubic equation having the form, $x^3 + 3A^2x = B$.

$$\text{Let } C = \sqrt{\frac{2B}{A^3} + 1} - 1. \quad \text{Then } x = \frac{2B}{A^2(C^2 + 12)} + \frac{AC}{4}.$$

It is required to show: (1) How this formula was derived; (2) under what conditions does it give exact results; (3) in general, what is its degree of approximation; (4) if possible, modify it so that it will always give exact results; (5) if it cannot be so modified, show why.

SOLUTION BY THE PROPOSER.

(1) The formula was derived by equating the volume of a spherical segment of one base and unknown altitude, x, to n times the volume of a sphere whose radius is the radius of the base of the spherical segment.

Thus, let r be the given radius of the base of the spherical segment of one base and x the unknown altitude. Then we have by hypothesis

$$\tfrac{1}{3}\pi x^2(3r - x) = n(\tfrac{4}{3}\pi r^3) = \tfrac{1}{2}\pi x r^2 + \tfrac{1}{6}\pi x^3.$$

Hence,

$$3rx^2 - x^3 = 4nr^3,$$

and

$$3r^2x + x^3 = 8nr^3;$$

whence, by adding the two equations and reducing,

$$r^2x + rx^2 = 4nr^3.$$

Whence

$$x = -\frac{r}{2} \pm \frac{r}{2}\sqrt{16n+1}.$$

Letting $r^2 = A^2$, $B = 8nr^3$, and $C = \sqrt{\frac{2B}{A^3}+1} - 1$, we have

$$x = \frac{AC}{2}. \tag{1}$$

But

$$\frac{B}{\frac{A^3C^3}{8} + \frac{3A^3C}{2}} = 1.$$

Hence,

$$x = \frac{AC}{2} \times \frac{B}{\frac{A^3C^3}{8} + \frac{3A^3C}{2}} = \frac{4B}{A^2(C^2+12)}. \tag{2}$$

From the last two values of x, we have, by addition and dividing by 2,

$$x = \frac{2B}{A^2(C^2+12)} + \frac{AC}{4}. \tag{3}$$

(2) This always gives exact values when $n = \frac{1}{2}$.

As an application, take the cubic equation,

$$x^3 + 12x = 1{,}120.$$

Here $A = 2$, $B = 1{,}120$, and $C = \sqrt{281} - 1$. Hence, from (1), $x = 15.7$; from (2), $x = 8.4$, and from (3), $x = 10.03$. Thus (3) gives a very close approximation.

Remark by the Editors.

The proposer does not answer (3), (4), and (5). The deductions in his solution result from the confusion of letters representing different quantities and using them to represent the same quantity without drawing the necessary conclusion therefrom. Thus, in the expression $\frac{1}{3}\pi x^2 \times (3r - x)$, the r here is the radius of the sphere from which the segment is taken, whereas in $\frac{1}{2}\pi r^2 x + \frac{1}{6}\pi x^3$, the r is the radius of the base of the segment. If these two expressions are equated as they are in the solution above, it follows that $x = r$ and therefore $n = \frac{1}{2}$. It thus follows that when $n = \frac{1}{2}$ the formula gives correct values for cubics of the proposed form. This seems to us to answer the remaining questions in his problem.

387. Proposed by H. C. FEEMSTER, York College, Nebraska.

Sum the following series:

$$(1)\ \sum_{n=1}^{n=\infty} \frac{1}{8^{n+1}n!};\quad (2)\ \sum_{n=1}^{n=\infty} \frac{1\cdot 3\cdot 5 \cdots 2n+1}{2^{2n+1}(2n-1)!};\quad (3)\ \sum_{n=1}^{n=\alpha} \frac{1}{2^{2n}n!}.$$

SOLUTION BY J. A. BULLARD, Worcester Polytechnic Institute.

By Maclaurin's Theorem, we have

$$f(x) = \sum_{n=0}^{n=\infty} \frac{x^n}{n!} D^n f(0), \tag{A}$$

where $D^n f(0) = [D^n f(x)]_{x=0}$, $0! = 1$ and $D^0 f(0) = f(0)$.

(1) Let $f(x) = e^x$. Then by (A),

$$\frac{e^x}{8} = \frac{1}{8}\sum_{n=0}^{n=\infty}\frac{x^n}{n!} = \frac{1}{8}\left\{1 + \sum_{n=1}^{n=\infty}\frac{x^n}{n!}\right\}. \qquad (B)$$

Letting $x = \frac{1}{8}$, and reducing, we readily have

$$\frac{e^{\frac{1}{8}} - 1}{8} = \sum_{n=1}^{n=\infty}\frac{1}{8^{n+1}n!}.$$

(2) $$\sum_{n=1}^{n=\infty}\frac{1\cdot 3\cdot 5\cdots 2n+1}{2^{2n+1}(2n-1)!} = \sum_{n=1}^{n=\infty}\frac{(2n+1)!}{2^{3n+1}(2n-1)!\,n!}$$

$$= \sum_{n=1}^{n=\infty}\frac{2n+1}{2^{3n}(n-1)!} = \frac{1}{8}\sum_{n=1}^{n=\infty}\frac{2n+1}{8^{n-1}(n-1)!}.$$

Let $f(x) = (2x+3)e^x$. Then $D^0f(0) = 3$, $D'f(0) = 5$, $\cdots$, $D^nf(0) = 2n+3$. By equation (A), we have

$$\frac{(2x+3)e^x}{8} = \frac{1}{8}\sum_{n=0}^{n=\infty}\frac{x^n}{n!}(2n+3).$$

Letting $x = \frac{1}{8}$, we have

$$\frac{(\frac{1}{4}+3)e^{\frac{1}{8}}}{8} = \frac{1}{8}\sum_{n=0}^{n=\infty}\frac{2n+3}{8^n n!}.$$

Replacing n by $n-1$, we have

$$\frac{13e^{\frac{1}{8}}}{32} = \frac{1}{8}\sum_{n=1}^{n=\infty}\frac{2n+1}{8^{n-1}(n-1)!} = \sum_{n=1}^{n=\infty}\frac{1\cdot 3\cdot 5\cdots(2n+1)}{2^{2n+1}(2n-1)!}.$$

(3) Letting $x = \frac{1}{2^2}$ in equation (B) and multiplying by 8, we have

$$e^{\frac{1}{4}} - 1 = \sum_{n=1}^{n=\infty}\frac{1}{2^{2n}n!}.$$

Excellent solutions were received from A. M. HARDING, R. M. MATHEWS, G. N. BAUER, H. L. SLOBIN, and S. LEFSCHETZ.

Note.—The Proposer intended the sign in the denominator of (2) to be + instead of −. With that change his answer for (2) is $\frac{1}{2} - \frac{1}{4}\sqrt{2}$. EDITOR.

388. Proposed by JOSEPH V. COLLINS, Stevens Point, Wis.

On a certain typewriter there is a double scale as follows:

80··· 70···60···50···40···30···20···10··· 0
1··· 5···10···15···20···25···30···35···40

It is used to locate headings in the middle of the page. To space a heading, one sets the machine at the right stop and with the spacer counts out the number of letters and spaces in the heading. To the reading on the 40 scale where the carriage stops is added the reading of the right stop on the same scale. This number is the one on which to set the carriage pointer on the 80 scale to begin the heading. Show by algebra that the method is correct.

SOLUTION BY THE PROPOSER.

Let a = number of letters and spaces in heading.
b = number of spaces on 80 scale of right stop.

Then $\frac{80-a}{2}$ = number on 80 scale to begin heading. But by the method used this number is found as $\left\{\left(40-\frac{b}{2}\right)-\frac{a}{2}\right\}+\frac{b}{2}$.

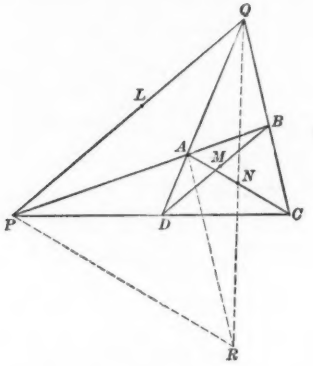
GEOMETRY.

418. Proposed by MRS. H. E. TREFETHEN, Waterville, Maine.

The difference of the squares of the two interior diagonals of a cyclic quadrilateral is to twice their rectangle as the distance between their mid-points is to the third diagonal.

SOLUTION BY C. N. SCHMALL, New York, N. Y.

Let $ABCD$ be a cyclic quadrilateral. Let the sides BA, CD meet in P, and the sides DA, CB in Q. Draw PQ. The resulting figure is a *complete quadrilateral* and PQ is the *third diagonal*. Let L, M, N, be the mid-points of the three diagonals. Then we have to prove that



$$\frac{\overline{AC}^2-\overline{BD}^2}{2AC\cdot BD}=\frac{MN}{PQ}.$$

It is well known that the points, L, M, N, are collinear. (See RUSSELL'S *Sequel to Elem. Geom.*, p. 154, ex. 5; or LACHLAN'S *Pure Geom.*, p. 93, § 151; or MILNE'S *Cross-Ratio Geom.*, p. 113, ex. 3.) Hence $MN = LN - LM$, or

$$\frac{MN}{PQ}=\frac{LN}{PQ}-\frac{LM}{PQ}. \tag{1}$$

Draw QN and prolong it to R, making $NR = QN$. Draw AR. Then $QCRA$

is a parallelogram. Hence QC is parallel to AR and

$$QC = AR. \tag{2}$$

Now, the triangles PAC, PDB, are similar; for $\angle PCA = \angle PBD$, both standing on the arc AD in a cyclic quadrilateral. Hence

$$AC : DB :: PA : PD. \tag{3}$$

Likewise, the triangles QCA, QDB are similar; and

$$AC : DB :: QC : QD. \tag{4}$$

From (3) and (4),

$$PA : PD :: QC : QD;$$

or

$$PD : QD :: PA : QC;$$

or, by (2),

$$PD : QD :: PA : AR. \tag{5}$$

Again,

$$\angle PDQ = \angle PAR, \tag{6}$$

for

$$\angle PDQ = 180° - \angle ADC = \angle ABC$$

and

$$\angle PAR = \angle ABC.$$

Hence, by (5) and (6) the triangles PDQ and PAR are similar; since they have an angle in each equal and the including sides proportional. Therefore,

$$PA : PD :: PR : PQ;$$

or

$$PA : PD :: 2LN : PQ. \tag{7}$$

By (3) and (7) we have

$$AC : BD :: 2LN : PQ. \tag{8}$$

In a similar way, by drawing QM and prolonging it to S (say), making $MS = QM$, and then drawing PS, we can show that

$$AC : BD :: PQ : 2LM. \tag{9}$$

From (8) and (9),

$$\frac{LN}{PQ} = \frac{1}{2}\frac{AC}{BD}, \qquad \frac{LM}{PQ} = \frac{1}{2}\frac{BD}{AC}.$$

Substituting these values in (1), we have

$$\frac{MN}{PQ} = \frac{1}{2}\left(\frac{AC}{BD} - \frac{BD}{AC}\right); \qquad \text{or} \qquad \frac{MN}{PQ} = \frac{\overline{AC}^2 - \overline{BD}^2}{2AC \cdot BD}.$$

CALCULUS.

333. Proposed by C. N. SCHMALL, New York City.

Evaluate $\int\int\int \sqrt{\frac{1-(x^2+y^2+z^2)}{1+x^2+y^2+z^2}}\, dx\, dy\, dz$, where $x^2 + y^2 + z^2 < 1$.

SOLUTION BY THE PROPOSER.

The given integral evidently depends on the distance of the point (x, y, z) from the origin. As the integration is to be taken for all values of x, y, and z within a sphere of unit radius, center at the origin, the integral may be put into the form

$$u = 4\pi \int_0^1 \sqrt{\frac{1-r^2}{1+r^2}} \cdot r^2 dr,$$

in which the element of volume is a thin spherical shell included between the spheres of radii r, $r + dr$.

Now put $r = \sin\phi$. Then we have

$$u = 4\pi \int_0^{\frac{\pi}{2}} \frac{\cos^2\phi \sin^2\phi}{\sqrt{1+\sin^2\phi}}\, d\phi = 4\pi \int_0^{\frac{\pi}{2}} \frac{\frac{1}{4}\sin^2 2\phi}{\sqrt{1+\sin^2\phi}}\, d\phi \tag{1}$$

$$= 4\pi \int_0^{\frac{\pi}{2}} \left(\frac{\frac{1}{4}\sin^2 2\phi}{\sqrt{1+\sin^2\phi}} + \cos 2\phi \sqrt{1+\sin^2\phi} \right) d\phi - 4\pi \int_0^{\frac{\pi}{2}} \cos 2\phi \sqrt{1+\sin^2\phi}\, d\phi$$

$$= 4\pi \left[\tfrac{1}{2} \sin 2\phi \sqrt{1+\sin^2\phi} \right]_0^{\frac{\pi}{2}} - 4\pi \int_0^{\frac{\pi}{2}} \cos 2\phi \sqrt{1+\sin^2\phi}\, d\phi$$

$$= -4\pi \int_0^{\frac{\pi}{2}} \cos 2\phi \sqrt{1+\sin^2\phi}\, d\phi = 4\pi \int_0^{\frac{\pi}{2}} (2\sin^2\phi - 1)\sqrt{1+\sin^2\phi}\, d\phi$$

$$= 4\pi \int_0^{\frac{\pi}{2}} \frac{(1-2\cos^2\phi)(1+\sin^2\phi)\, d\phi}{\sqrt{1+\sin^2\phi}}$$

$$= 4\pi \int_0^{\frac{\pi}{2}} \frac{1-2\cos^2\phi+\sin^2\phi-2\sin^2\phi\cos^2\phi}{\sqrt{1+\sin^2\phi}}\, d\phi$$

$$= 4\pi \int_0^{\frac{\pi}{2}} \frac{1-2\cos^2\phi+\sin^2\phi}{\sqrt{1+\sin^2\phi}}\, d\phi - 2u \quad \text{(by equation (1) above).}$$

Hence

$$3u = 4\pi \int_0^{\frac{\pi}{2}} \frac{3\sin^2\phi - 1}{\sqrt{1+\sin^2\phi}}\, d\phi = 4\pi \int_0^{\frac{\pi}{2}} \left(3\sqrt{1+\sin^2\phi} - \frac{4}{\sqrt{1+\sin^2\phi}} \right) d\phi$$

$$= 4\pi \int_0^{\pi} 3\sqrt{1+\sin^2\phi}\, d\phi - 4\pi \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1+\sin^2\phi}}.$$

Putting $\pi/2 - \phi$ for ϕ, we obtain

$$3u = 12\sqrt{2}\cdot\pi \int_0^{\frac{\pi}{2}} \sqrt{1-\tfrac{1}{2}\sin^2\phi}\, d\phi - 16\sqrt{2}\cdot\pi \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1-\frac{1}{2}\sin^2\phi}}.$$

These are complete elliptic integrals of the second and first kinds respectively, and can be readily evaluated. See Peirce's *Short Table of Integrals*, pp. 118–119, and Bromwich's *Infinite Series*, p. 162, ex. 6.

Using the notation employed in the former, we have

$$u = \tfrac{1}{3}\{12\pi\sqrt{2}E(\sqrt{\tfrac{1}{2}},\ \phi) - 16\pi\sqrt{2}F(\sqrt{\tfrac{1}{2}},\ \phi)\}.$$

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

188. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Solve the congruence $3x^2 + 4x + 5 \equiv 0 \pmod{20}$.

SOLUTION BY WALTER C. EELLS, Whitworth College.

By inspection, solutions of the two congruences,

$$3x^2 + 4x + 5 \equiv 0 \pmod 4, \tag{1}$$

$$3x^2 + 4x + 5 \equiv 0 \pmod 5, \tag{2}$$

are $x = \pm 1$ for (1), and $x = 0, 2$ for (2).

Then we have the four systems of linear congruences,

$$(3)\ \begin{cases} x \equiv 1 \pmod 4, \\ x \equiv 0 \pmod 5, \end{cases} \qquad (4)\ \begin{cases} x \equiv 1 \pmod 4, \\ x \equiv 2 \pmod 5, \end{cases}$$

$$(5)\ \begin{cases} x \equiv -1 \pmod 4, \\ x \equiv 0 \pmod 5, \end{cases} \qquad (6)\ \begin{cases} x \equiv -1 \pmod 4, \\ x \equiv 2 \pmod 5. \end{cases}$$

To solve system (3), substitute $x = 1 + 4y$ from first congruence in second congruence,

$$1 + 4y \equiv 0 \pmod 5;$$

whence

$$y = 1 + 5k,$$

and

$$x = 1 + 4y = 1 + 4(1 + 5k) = 5 + 20k.$$

Hence, $x \equiv 5 \pmod{20}$, yielding one solution of given congruence. Similarly, systems (4), (5), (6) yield $x = 17, 15, 7$, giving altogether four independent roots of the congruence $3x^2 + 4x + 5 \equiv 0 \pmod{20}$.

Also solved by H. C. FEEMSTER, LOUIS CLARK, S. LEFSCHETZ, E. B. ESCOTT.

No solution of 189 has been received.

MISCELLANEOUS QUESTIONS.

EDITED BY R. D. CARMICHAEL.

When it was necessary to make ready the manuscript for this issue our readers had not yet had time to reply to our first questions, proposed in the November number. It is believed that some replies to those questions may be inserted in the issue next following this one. In the meantime we have received two other interesting questions. They are printed here; and the attention of our readers is directed to them.

QUESTIONS.

3. In connection with the theory of the conduction of electricity through gases, one is led to the differential equation

$$y\frac{d^2y}{dx^2} + a\left(\frac{dy}{dx}\right)^2 + b\frac{dy}{dx} + cy + d = 0,$$

where a, b, c, d are constants. For unrestricted values of a, b, c, d the solution of this differential equation presents peculiar difficulties, the series solutions obtained by the customary methods having (apparently) too small a range of convergence to be satisfactory from the point of view of electrical theory. The general solution of this equation is wanted in case it can be found. If no general solution is obtained for unrestricted a, b, c, d, it is desirable to know special values of a, b, c, d or special relations among a, b, c, d which make it possible to find the general solution; and this solution is desired in each case.

We shall be glad to receive an answer to any part of this question in case a complete answer is not found.

4. In analytic geometry classes we teach about six formulas for the straight line and often we drill pupils in their use only with formal exercises. Mr. R. M. MATHEWS, of Riverside, California, proposes a collaboration through the MONTHLY by means of which a set of problems of interest in themselves shall be collected for these drill exercises. Our readers are requested to send to the editor of this department such problems, classified under headings suggesting the formula for which each problem is useful, the headings being as follows: two point form, slope point form, slope intercept form, two intercept form, normal form, general form.

As examples of the sort of problem desired we have the following, suggested by Mr. Mathews:

Two point form.—A linear relation connects degrees centigrade with degrees Fahrenheit. Find this relation if 50° F. = 10° C. and − 4° F. = − 20° C.

Slope intercept form.—The electrical resistance of annealed copper wire is 9.59 ohms per mil-foot at 0° C., from which point it increases with the temperature in the proportion of 403 per thousand. Express the resistance in terms of temperature centigrade.

NOTES AND NEWS.

UNDER THE DIRECTION OF FLORIAN CAJORI.

The winter meeting of the Chicago Section of the American Mathematical Society wil held at Chicago on December 26–27, 1913.

The October number of the *Popular Science Monthly* contains an article on the Fourth Dimension, by Professor SAMUEL M. BARTON, of the University of the South.

The firm of Gauthier-Villars is undertaking the publication of the complete works of the late HENRI POINCARÉ, under the direction of the Paris Academy of Sciences and the French Minister of Public Instruction.

H. Grassmann, of the University of Giessen, the second volume of whose *Projektive Géometrie* has just been brought out by Teubner, is a son of the former professor in the gymnasium at Stettin, who is celebrated as the founder of the Ausdehnungslehre.

The San Francisco section of the American Mathematical Society held a regular meeting on October 25 at Stanford University. The following names appeared on the program: Professors L. M. Hoskins, M. W. Haskell, W. C. Eells, G. A. Miller, W. A. Manning and Mr. B. A. Bernstein.

Mr. A. C. Johnson, formerly of Cincinnati, Ohio, has recently become principal of the high school at Hopedale, Mass.

Dr. N. J. LENNES, of the department of mathematics at Columbia University, has accepted the position of professor of mathematics at the University of Montana, where he succeeds Professor L. C. Plant, who goes to the Michigan Agricultural College as head of the department of mathematics there.

Owing to the important rôle that questions of psychology play at the present time in discussions of mathematical teaching, a booklet on *Psychologie und mathematischer Unterricht*, prepared by D. Katz, under the editorship of Professor FELIX KLEIN, will be of general interest.

On September 6, 1913, occurred the death of Professor LUCIAN AUGUSTUS WAIT, of Cornell University, at the age of 67 years. After graduating at Harvard in 1870, he served on the mathematical faculty of Cornell until 1910, when he became professor emeritus. He is well known as one of the authors of the Oliver Wait & Jones series of mathematical texts.

Professor CHARLES GREENE ROCKWOOD, of Princeton, died on July 2, 1913, at the age of 71 years. He was a Yale graduate. He began his career as a teacher of college mathematics at Bowdoin and Rutgers, and during 1877–1905 was professor of mathematics at Princeton. Since 1905 he was professor emeritus. His early teaching embraced also natural philosophy and astronomy. His wide scientific interests are evident from the large number of scientific societies of which he was a member. He took part in the Princeton eclipse expedition to Colorado in 1878 and was a contributor on seismology to the *American Journal of Science*.

Recommendations on the unification of mathematical notations are contained in Heft 17 issued by the German committee on the teaching of mathematics and science. The recommendations are reprinted in Schotten's *Zeitschrift* for September, 1913. It will be remembered that a report on the same subject was made by an American committee under the chairmanship of L. P. Jocelyn, of Ann Arbor.

Dr. ALEXANDER MACFARLANE, whose death was announced in the last number of the MONTHLY, wrote to one of the editors less than two weeks before his death with regard to the history of the exponential and logarithmic concepts which appeared in the MONTHLY. He expressed himself as follows: "These concepts play the leading parts in the higher developments of vector-analysis; therefore

it behooves every vector-analyst to make himself thoroughly familiar with the history."

Plans are being made to commemorate the seventh centenary of Roger Bacon's birth at Oxford in July, 1914, when a statue of Bacon will be unveiled, and addresses will be given by distinguished scholars. There will be issued a memorial volume of essays dealing with various aspects of Roger Bacon's work. It is also proposed to prepare an edition of Bacon's writings, including various MSS. not hitherto published. To secure funds for such an edition it is proposed to organize a Roger Bacon Society.

At the University of Michigan the following men have been appointed as instructors in mathematics: T. Fort, Ph.D. (Harvard) 1912, student at Göttingen, 1912–1913; L. J. Rouse, A.B. (Princeton) 1908, Scholar at Univ. of Pennsylvania, 1912–1913; C. H. Forsythe, A.M. (Illinois) 1910, graduate student at Cornell, Illinois, and Michigan; B. Libby, B.S. (Princeton) 1911, graduate student at Wisconsin, 1911–1913. At the October meeting of the Board of Regents Assistant Professor T. R. Running was promoted to a junior professorship in mathematics and Dr. Clyde E. Love, from an instructorship to an assistant professorship. Assistant Professor Peter Field was promoted to a junior professorship at a previous meeting. Mr. L. A. Hopkins is absent on leave, as Fellow in the University of Chicago, and Mr. Carl Coe as Scholar at Harvard University.

According to *Science*, W. H. YOUNG, professor of mathematics in Liverpool University, has been appointed professor of mathematics in the University of Calcutta, for the purpose of organizing there a new school of mathematics. As the duties of the post require his residence in India only from November till March, he will retain his professorship in Liverpool University.

In the *Annaes da academia polytechnica do Porto* (Volume VIII, 1913) Yoshio Mikami gives an account of a seventeenth century Portuguese scientist who went to Japan as a Christian missionary, but, to escape persecution by the Shogunate, was compelled to forsake his faith and render service to the authorities. He assumed a Japanese name. His real name is still unknown. In 1650 he translated into Japanese an astronomical treatise. Mikami concludes that Japanese scientists of the seventeenth century became acquainted with European astronomy not so much through the Chinese, as is sometimes believed, but mainly through direct contact with Europeans. In 1687 the Shogunate prohibited the importation into Japan of 38 works compiled in China from European sources, for the reason that they bore on "the accursed religion of Christianity." Among these treatises were a translation of Euclid, a treatise on arithmetic, and some works in astronomy and surveying.

The Annual Report of the Smithsonian Institution for 1912, published in 1913, contains the address on "Molecular theories and mathematics," delivered by Professor ÉMIL BOREL, of Paris, on the occasion of the inauguration of the Rice Institute, October 10 to 12, 1912. This volume contains three other articles of special mathematical interest, as follows: "The connection between the ether and

matter," by HENRI POINCARÉ; "Modern mathematical research," by G. A. MILLER; and "HENRI POINCARÉ, his scientific work, his philosophy," by CHARLES NORDMANN.

The central committee of the International Commission on the Teaching of Mathematics announces a Congress of the Commission to be held at Paris, April 6 to 8, 1914. The details of the meeting will be announced later. The arrangements at Paris were in charge of Professor Carlo Bourlet, who died on the twelfth of August, from the effects of an accident, and consequently the completion of the program is delayed. At present, it can be definitely announced that two public sessions will be devoted to the consideration of reports on the following two questions:

A. The results obtained in the introduction of the differential and the integral calculus into secondary instruction. General Reporter, PROFESSOR E. BEKE, Budapest.

B. The place and function of mathematics in advanced technological instruction. General Reporter, PROFESSOR P. STAECKEL, Heidelberg.

These reports will seek to summarize conditions and tendencies as to these questions throughout the world, and will be based on replies received from the various nations in response to a questionnaire that has been prepared on each topic. It is planned, further, to hold a meeting of the Commission at Munich, in the summer of 1915, and a final meeting in connection with the Sixth International Congress of Mathematicians at Stockholm, in 1916.

It is to be noted that the term "secondary instruction," as used in the foregoing announcement, refers to instruction given in the last two years of schools like the German *gymnasia*, and would be comparable in the United States to the fifth and sixth year in a six-year high school or to the freshmen and sophomore courses in our colleges. The chief interest to us in this report will be its bearing upon the general question as to whether it would be desirable to teach the elements of the calculus much earlier than we do, for instance, in connection with college algebra and analytic geometry.

Dr. H. F. Baker's presidential address before the mathematical and physical science section of the British Association for the Advancement of Science, was entitled "The Place of Pure Mathematics." It is published in full in *Science* of September 12, 1913. In the course of the introduction he says: "What, then, is this subject? What can it be about if it is not primarily directed to the discussion of the laws of natural phenomena? What kind of things are they that can occupy alone the thoughts of a life-time? I propose now to attempt to answer this, most inadequately, by a bare recital of some of the broader issues of present interest."

Under "Precision of Definitions" Dr. Baker points out that "it is a constantly recurring need of science to reconsider the exact implication of the terms employed," and illustrates this by reference to "those series which Fourier used so boldly, and so wickedly, for the conduction of heat." "Like all discoverers, he took much for granted. Precisely how much is the problem. This problem

has led to the precision of what is meant by a function of real variables, to the question of the uniform convergence of an infinite series, as you may see in early papers of Stokes, to new formulation of the conditions of integration and of the properties of multiple integrals, and so on. And it remains still incompletely solved."

Dr. Baker then refers to the calculus of variations, and is led to speak of the stability of the solar system. "For those who can make pronouncements in regard to this I have a feeling of envy; for their methods, as yet, I have quite another feeling. The interest of this problem alone is sufficient to justify the craving of a pure mathematician for powerful methods and unexceptionable rigor." He then proceeds to philosophical considerations on non-euclidean geometry, the theory of groups, algebraic functions, functions of complex variables, and the theory of numbers. "Each of those I have named is large enough for one man's thought; but they are interwoven and interlaced in indissoluble fashion and form one mighty whole, so that to be ignorant of one is to be weaker in all. I am not concerned to depreciate other pursuits, which seem at first sight more practical; I wish only, indeed, as we all do, it were possible for one man to cover the whole field of scientific research; and I vigorously resent the suggestion that those who follow these studies are less careful than others of the urgent needs of our national life."

The "southerly deviation of falling bodies" has recently been re-examined mathematically by three Americans. Dr. W. H. ROEVER, of Washington University, has contributed articles to the *Transactions* of the Amer. Math. Soc. (Vol. 12, 1911, 335–353; Vol. 13, 1912, 469–490), in which under the assumption of a distribution of revolution; that is, assuming the potential function of the earth's gravitational field of force to be of the form $f(r, z)$, where r is the distance of a general point from the earth's axis of rotation and z is that from a fixed plane perpendicular to the axis, he obtains a formula for the southerly deviation which reduces, when g is taken constant, to the formula given by Gauss. Roever showed also that local irregularities in the earth's gravitational field of force caused by mountains and mineral deposits may influence the southerly deviation to the extent of from $-$ 16 times to $+$ 40 times the amount which is given by the formula of Gauss for the southerly deviation.

The theory of the problem of the southerly deviation is reconsidered by President R. S. Woodward of the Carnegie Institution (*Astronomical Journal*, Vol. 28, 1913, 17–29; *Science*, N. S., Vol. 38, 1913, 315–319), who asserts that the problem has "hitherto been inadequately treated by reason of neglect of the effect of the square of the angular velocity of the earth and other terms of the same order, and by equal neglect of the effect of difference between the geocentric latitude of the point of departure of the body and the geographical latitude of the horizontal plane to which the body falls. Failure to consider these effects has led to erroneous conclusions in respect to the meridional deviation of the body, the real deviation being, in fact, opposite to that derived by Gauss and most later investigators." According to Woodward's deductions, "for a fall of

10 seconds, or 490.24 meters (in vacuo) in latitude 45°, the meridional deviation would be 3.03 cm. and the easterly deviation 16.85 cm." Most experiments carried on in the northern hemisphere have shown a very slight deviation, not to the north, as Woodward's formula demands, but toward the south. Says Woodward, "I consider it quite impractical to make any conclusive experiments on the deviation of spheres falling in air."

A third treatment of freely falling bodies was presented on October 25, 1913, by Professor L. M. HOSKINS, of Stanford University, before the San Francisco section of the American Mathematical Society. According to the abstract of the paper, Professor Hoskins referred the motion, as one of the simplest methods of studying the problem, "to axes fixed in the earth, making use of the relation between the accelerations of a particle referred to fixed and moving axes. . . . The question whether this field would cause a deviation northward or southward from a plumb-line suspended from the point where the body starts, is closely connected with the question of the curvature of the lines of force in this field; the path would lie on the convex side of that line of force passing through the position of rest, while the plumb-line would hang on the concave side. So far as the effect of this field is concerned, the body will fall on that side of the plumb-line toward which the lines of force are convex. Analysis indicates that the convexity is toward the equator."

This issue brings to a close the first year of the MONTHLY under its new auspices. The question whether a journal devoted primarily to the collegiate field of mathematics would be appreciated and supported has been answered in the affirmative to a gratifying extent. The subscription list has increased continuously throughout the year, the acceleration being greatest during the past three months. There seems to be every indication that this condition will continue and that the MONTHLY may, within another year, become self-sustaining, even at the very low subscription price at which it is offered and with the very high degree of mechanical excellence at which it is maintained.

All present subscribers will materially contribute to the cause by promptly sending their renewals to the Treasurer without waiting for a bill from him. A renewal blank is enclosed with this issue. In the case of all renewals received before January 5, 1914, acknowledgment will be indicated by the date on the address label of the January issue.

MANAGING EDITOR.

Errata. In addition to the corrections noted on pages 104, 210, 258, the word *k-triple* in line 7 of Professor Dickson's article on page 84 should read *k-tuple*. Also in the second note on page 292, the name should be T. N. HAUN.

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FLORIAN CAJORI
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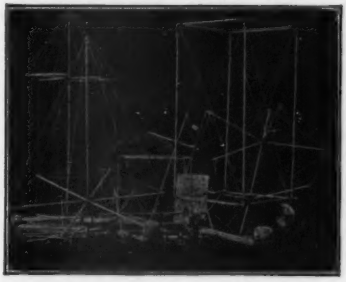

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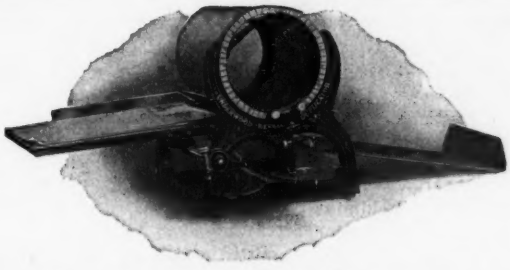

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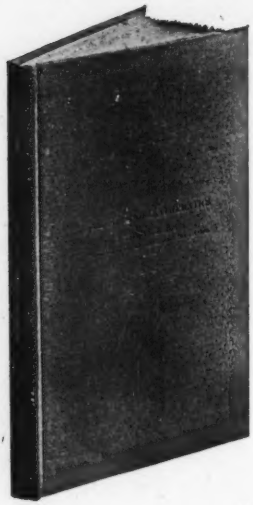

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